Liquidity Freezes Under Adverse Selection*

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Abstract

This paper analyses how adverse selection prevents liquidity from flowing from liquid to illiquid firms, thus impairing the transmission mechanism of policy. Contrary to the results in the literature, simply increasing the availability of liquidity does not solve the adverse selection problem. When there are aggregate shocks, authorities face a policy dilemma if their single policy tool is to manipulate the price of liquidity. We consider alternative policies which address the problem in a

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time-consistent fashion.

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1 Introduction

The effective implementation of monetary policy requires that the liquidity injected by the central bank flow throughout the economy to those firms which need it. Yet, the financial crisis and its aftermath provide many reminders that liquidity conditions can vary substantially across firms.

Such heterogeneity in liquidity conditions is hard to reconcile with the existence of sophisticated financial markets which provide a variety of instruments to insure against liquidity shocks. If firms are able to insure against these shocks, then liquidity will flow freely across the economy and only the aggregate amount of available liquidity should matter. It would seem to follow that a liquidity crisis can easily be stopped by flooding financial markets with liquidity, which will find its way to those individuals who most need it.

The free flow of liquidity is a characteristic of models such as the framework developed in Holmström and Tirole (1998, 2013). To address this limitation, we extend their model by adding adverse selection. In this extended model, we are able to analyze the conditions in which liquidity dries up in financial markets, and describe possible liquidity policies.

We find that adverse selection distorts financial contracts in ways which discourage insurance against liquidity shocks. Institutions voluntarily self-ration their liquidity insurance, since they would be perceived as low quality if they held more liquidity. Ex post, the equilibrium allocation leads to the inability to transfer resources among firms and to projects being liquidated. Flooding the market (ex ante) with liquidity at subsidized rates does not solve the problem because of the stigma associated with liquidity holdings. There is no role for the ex post provision of liquidity, except for bailing out institutions.[1]

[1] Heider, Hoerova, and Holthausen (2015) also analyze liquidity policy in the presence of asymmetric

Liquidity freezes in the aftermath of the financial crisis The events of the past decade provide many examples of situations in which liquidity does not flow freely.

- After the bankruptcy of Lehman Brothers, interbank markets stopped functioning well. The European Central Bank broadened the range of eligible collateral in its open market operations so as to offset the shortage of good collateral. As a result, many banks substituted from the secondary market to the primary one, leading the central bank to effectively replace the interbank market in the allocation of liquidity among banks.
- During the crisis, the Federal Reserve set up an emergency program to buy corporate short term debt with the aim of supporting the orderly functioning of the commercial paper market, as many corporations were no longer able to roll over their maturing commercial paper.
- The European sovereign debt crisis provoked heterogeneous liquidity conditions across the euro area. With the growing fear of a euro area break up, banks in core economies trimmed exposures to members under stress. Liquidity dried up in the periphery of the monetary union, leading local banks to scale back credit and sell their assets.

The above examples show a clear message: firms did not insure against liquidity shocks in advance so that, ex post, liquidity-rich firms are not reallocating liquidity to firms with a high shadow value for liquidity. A natural explanation for such a failure comes from differential information: agents with liquidity are unwilling to share it with other agents who are better informed about the risks involved.

information. They suggest that high quality institutions tend to find market liquidity too expensive and therefore refuse to participate, and that it is possible to encourage them back into the market by providing liquidity at a subsidized rate. Their analysis however treats the participants in liquidity markets as passive. Contractual arrangements and public information provide a set of available signals which convey information about the quality of institutions. For example, at the most basic, a choice by a bank about how much to borrow would provide a signal in the Heider, Hoerova, and Holthausen (2015) model thus overturning their result.

The Holmström and Tirole framework We examine the consequences of such adverse selection in the Holmström and Tirole (1998, 2013) framework. This is the natural baseline in markets with sophisticated financial institutions. Holmström and Tirole model liquidity as an insurance mechanism which allows firms the flexibility to make needed investments on short notice. Commercial banks, for instance, have long been in the business of liquidity management both for themselves and their clients, forecasting liquidity needs and providing for them in the most cost-effective way.

While financial institutions exist to manage liquidity needs of firms, the ability to obtain liquidity is constrained by limits to the pledgeability of their income. The amount of pledgeable income created by a firm's projects limits its ability to obtain external finance, and conditions both the size of the projects and the amount of liquidity insurance that can be obtained.

Firms want to acquire adequate liquidity in advance so as to avoid credit rationing at a later stage. In Holmström and Tirole framework, the second-best features full liquidity insurance, which means that liquidity will flow ex post to those firms which need it. The second-best allocation can be implemented, for example, by securing a credit line with a bank. Aggregate shocks can be accommodated by external provision of liquidity, through, for example, the monetary policy of a central bank.

When we add adverse selection to this framework, it impedes the efficient flow of liquidity. Purchasers of liquidity insurance have an informational advantage over providers because they know their own true risk types.

When liquidity is dearer to low quality institutions, liquidity demand becomes a tool for signaling the type and thus avoid paying a lemons premium. In the extreme case, the high quality institutions renounce liquidity insurance altogether, thus leading to a liquidity freeze. Ex post, the shadow value of liquidity is different across socially useful projects and liquidity-short firms must liquidate their projects. Indiscriminate liquidity provision does not encourage these institutions to obtain any insurance.

Plan of the paper The underlying assumptions of our model are described in Section 2. We start with the analysis of the optimal liquidity choices of the firm when there is perfect information about the type of the entrepreneur. We then consider the case with adverse selection. Good entrepreneurs distort their contracts in order to distinguish themselves from bad entrepreneurs in financial markets, thus guaranteeing cheaper funding.

Distortions are more serious when bad entrepreneurs are more efficient, as it becomes more difficult for good entrepreneurs to signal their type. We obtain two liquidity regimes. In the first regime, good firms can signal their type while obtaining partial insurance against liquidity shocks. Some liquidity flows from liquidity-long to liquidity-short firms, but liquidity-short firms inefficiently downsize their projects. In the second regime, there is a liquidity freeze—liquidity does not flow ex post and liquidity-short firms close down their projects. The analysis is carried out in Section 3 for idiosyncratic shocks and Section 4 for aggregate shocks.

Section 5 considers government policies. Increasing liquidity does not solve the adverse selection problem. In particular, the aggregate shocks case exhibits a key dilemma faced by those authorities whose single policy tool is manipulating the price of liquidity. On the one hand, flooding the market with liquidity so as to set the liquidity premium equal to zero does not solve the adverse selection problem. On the other hand, setting a positive liquidity premium may eliminate the adverse selection problem and lead the economy to the symmetric information equilibrium where entrepreneurs are fully insured against liquidity shocks, but at the cost of setting an inefficient low level of investment. We consider alternative policies which address the adverse selection problem in a time-consistent fashion.

Literature review The theoretical literature on the institutional demand for liquidity divides naturally into two camps, dealing respectively with ex ante and ex post provision of liquidity. Models which examine ex post liquidity tend to start from market incompleteness, assuming the inability to provide liquidity insurance ahead of time. Allen, Carletti, and Gale (2009) suggest that the central bank should stabilize interest rates through ex post open market operations, as aggregate uncertainty about liquidity demand leads to volatile interest rates, which is inefficient because it leads to volatile consumption for risk averse consumers. Confronting this view, Freixas, Martin, Skeie (2011) consider a different set of liquidity shocks and show that the optimal ex post interbank rate should be low during liquidity crises so as to facilitate the redistribution of liquidity.

Freixas and Holthausen (2005) consider peer monitoring in a model in which cross-border information about banks is less precise than home country information, and show that there is segmentation in the uninsured interbank market. Heider, Hoerova, and Holthausen (2015) propose a model of interbank borrowing and lending with adverse selection. Their model features separating and pooling equilibria which resemble the different phases of the malfunctioning of the European interbank during the 2007-09 crisis. There are key differences with our paper: first, they assume the amount of borrowed funds is not observable, which is plausible in the interbank market but unlikely in other markets. Instead, we emphasize that borrowed funds can provide a signal about the type of the entrepreneur and characterize the resultant equilibria. Unlike them, we find that it is not possible to encourage high quality institutions back into the market by providing liquidity at a subsidized rate. Second, they consider ex post counterparty risk and liquidity provision, whereas we consider ex ante adverse selection with time-consistent liquidity provision. Philippon and Skreta (2012) and Tirole (2012) consider the optimal design of ex post bailouts when adverse selection causes liquidity freezes, taking into account the stigma attached to government programs and the change in the terms of trade associated with asset purchases by the public sector.

Thus our model falls into the second camp, concerned with the ex ante provision of liquidity. These models assume institutions are able to obtain liquidity insurance, as for example in Holmström and Tirole (1998, 2013). When it comes to economic policy, these models emphasize preemptive time-consistent strategies. In a model with aggregate liquidity shocks, Farhi and Tirole (2012) show that firms privilege leverage and scale when they anticipate authorities will bail them out, even though firms would choose to fully insure against liquidity shocks if there were no government. Kahn and Wagner (2012) examine both ex ante and ex post liquidity shortages; the role of the central bank in a crisis depends crucially on the type of liquidity shortage experienced. Freixas and Jorge (2008) distinguish the pledgeable income of the firm from the pledgeable income of the bank; rationing in the interbank market causes a shortage of funding among bank dependent borrowers.

The empirical literature documents features present in our model, such as the demand for outside and inside liquidity (Krishnamurthy and Vissing-Jorgenssen 2012, and Krishnamurthy and Vissing-Jorgenssen 2013, respectively), the importance of collateral (Gorton and Metrick 2012), the existence of adverse selection in liquidity markets (Afonso, Kovner and Schoar 2011, Covitz, Liang and Suarez 2013), differences in the shadow value of liquidity across financial institutions (Cassola, Hortaçsu and Kastl 2013), the difficulties of central banks to improve the allocation of liquidity when there is adverse selection (Brunetti, Filippo and Harris 2011), and moral hazard caused by the anticipated policy reaction (Dam and Koetter 2012).

2 The Model

We start with the Holmström and Tirole (1998, 2013) model. There are three dates $t = 0, 1, 2$, a single good and no discounting. All agents are risk neutral.

There is a continuum of entrepreneur-owned firms each possessing a project. At date 0, each firm chooses the scale of the project I, measured by the amount of input required at that date. At date 1, each firm suffers a liquidity shock which can take one of two values: either low, ρ_L, or high, ρ_H. The value of the liquidity shock determines how much additional capital needs to be invested per unit at date 1 for the project to continue. It is possible to continue at a smaller scale than I, and the continuation scale is denoted i with $0 \leq i \leq I$. Thus if the project continues at scale i the total investment input equals $I + i\rho_L$ when the liquidity shock is low, and $I + i\rho_H$ when the liquidity shock is high. Firms have no alternative projects, so funding is only useful to cope with liquidity shocks. The realization of the shock for each firm is publicly observable.

Returns are realized at date 2, and there are no returns from the portion of the project that is not carried forward. The project yields a pledgeable return $\rho_0 i$. The firm can credibly commit in advance to pay pledgeable income to outsiders. In addition the project yields an illiquid private return $(\rho_1 - \rho_0)\, i$ to the entrepreneur. Entrepreneurs are protected by limited liability; thus there is no way to force an entrepreneur to pay more than $\rho_0 i$ in period 2. Throughout we assume

$$0 \leq \rho_L < \rho_0 < \rho_H < \rho_1.$$

These inequalities have the following interpretations: The low liquidity shock would not require pre-arranged financing; at date 1 the firm could pay for investing with credible promises to repay in period 2. However, the high liquidity shock is not self-financing. Since the initial investment I is a sunk cost as of period 1, it is efficient to continue the project ex post. Let f and $1 - f$ denote the probabilities of a low and a high liquidity shock.

All firms have a date 0 endowment $A > 0$, and no endowments at dates 1 and 2. They need $I - A$ in external funds to be able to invest at scale I.

Outside investors/consumers are competitive, and are willing to lend at a zero interest rate. They have no pledgeable wealth and so cannot borrow against promises to repay in the future. For convenience there is also an intermediary sector which trades in financial contracts with other agents in the economy; it owns no assets of its own and is competitive. While it can be thought of as either a banking or an insurance system, it basically serves as an accounting device in the model.

To the Holmström and Tirole model we add heterogeneity in firms' expected liquidity needs. There is a measure α of good firms (denoted G) and a measure $1-\alpha$ of bad firms (denoted B), with $0<\alpha<1$. The two types of firms are indistinguishable, and differ only in their probabilities of liquidity shocks. For good firms $f=f_G$ and for bad firms $f=f_B$, with $f_G>f_B$.

We impose a set of conditions on the returns of the good and the bad projects. Let $\overline{f}=\alpha f_G+(1-\alpha)\,f_B$ or, in other words, $\overline{f}$ is a population average.

Assumption 1

$$\rho_0<\frac{1+f_G\rho_L}{f_G} \tag{1a}$$

$$\rho_1<\min\left\{1+f_B\rho_L+(1-f_B)\,\rho_H,\frac{1+f_B\rho_L}{f_B}\right\} \tag{1b}$$

$$\frac{1+\overline{f}\rho_L}{\overline{f}}<\rho_1. \tag{1c}$$

The right-hand side in expression (1a) represents the expected cost of one unit of the good project (adjusted by the probability of completing the project) when the project is abandoned in the high shock state. The inequality implies that good projects are not self-financing (even when the project is continued in both states). Expression (1a) implies *a fortiori* that the average project is not self-financing and, from the social point of view, continuing in both states is better than continuing only in the low shock state, that is

$$\rho_0 < 1 + \overline{f}\rho_L + \left(1 - \overline{f}\right)\rho_H < \frac{1 + \overline{f}\rho_L}{\overline{f}}. \tag{2}$$

Expression (1b) states that bad firms are not socially useful, and outside investors will not finance bad firms if they identify them. The only possibility for bad entrepreneurs of getting finance is a pooling equilibrium, in which they mimic the good entrepreneurs. Expression (1c) states that the average project is socially useful, and implies *a fortiori* that the good project is socially useful.

We consider two cases. In Section 3 the individual firms' liquidity shocks are uncorrelated, and there is no aggregate risk. In Section 4 there are aggregate liquidity shocks, and shocks of firms of the same type are perfectly correlated. In the latter case, as shown in Holmström and Tirole, it will be of use to consider the possibility that the government provides "outside liquidity" by issuing bonds.

Government bonds are risk free assets issued at $t = 0$, which pay one unit of the good at date $t = 1$.

The price of government bonds at date 0 is q, and $q \geq 1$ since consumers are indifferent between consumption in dates 0 and 1 (if $q < 1$, consumers would demand an infinite amount of government bonds); the value of q may be greater than one, since the income of consumers is not pledgeable and they cannot supply liquidity. This asset enables agents to transfer wealth across periods, and is thus one means of providing liquidity to the corporate sector.

An entrepreneur's *contract* specifies the initial investment and the continuation scales, as well as the payments to be made or received for each date and liquidity shock.

An *equilibrium* specifies choices of contracts bought and sold by the agents in the economy and the price of government bonds.

It turns out that in all equilibrium contracts, the entrepreneur retains exactly the nonpledgeable income from the project.

Lemma 1 *In any equilibrium contract* (*i*) *the pledgeable income of the project is appropriated by outside investors, and* (*ii*) *entrepreneurs keep the nonpledgeable income.*

Since entrepreneurial capital has a higher rate of return than the cost of outside capital, it is optimal for the entrepreneur to commit all of the firm's pledgeable income to the outside investors so as to make project scale as great as possible, and to only keep the illiquid portion of the return (the nonpledgeable return associated with $\rho_1 - \rho_0$). This specification of payments maximizes the return on the entrepreneur's initial assets A.

3 Idiosyncratic liquidity shocks

This section describes the optimal contract under symmetric information about the quality of firms, and the market equilibrium under adverse selection.

With idiosyncratic liquidity shocks, the system is able to generate sufficient liquidity internally, so that there is no need for an outside source of liquidity (in contrast to the aggregate liquidity shocks case of Section 4). Still, the corporate sector is unable to distribute the liquidity internally, so that under adverse selection we obtain two regimes depending on the efficiency of bad firms—as measured by f_B. In the first regime, firms with low liquidity shocks channel their excess liquidity to firms with liquidity shortages, but such liquidity insurance is insufficient when compared with the symmetric information allocation. In the second regime, financial markets freeze and illiquid firms are terminated. In both regimes, financial markets are unable to redistribute excess liquidity efficiently, exposing firms to refinancing problems in case of a bad shock.

3.1 Symmetric information about the type of the firm (no adverse selection)

Suppose the continuation scale is i_L when the liquidity shock is low, and i_H when the liquidity shock is high, with $0 \leq i_L, i_H \leq I$. If the entrepreneur's type were publicly known, the optimal contract for the good entrepreneur is as in Holmström and Tirole, and the bad entrepreneur would not get any funding.

Proposition 1 *(Symmetric information contracts). Under symmetric information about the quality of the firms, there is a unique equilibrium in financial markets. In this equilibrium, $q = 1$, and the good entrepreneurs invest*

$$I^* = \frac{A}{1 - f_G\left(\rho_0 - \rho_L\right) - \left(1 - f_G\right)\left(\rho_0 - \rho_H\right)}$$

and set $i_H = i_L = I$. Bad entrepreneurs' projects are not funded.

The optimal policy for the good entrepreneur trades off the scale of the initial investment against the ability to withstand high liquidity shocks. Assumption 1 (expression 1a) guarantees that it is optimal to continue the project under a high liquidity shock. A good entrepreneur will anticipate that he cannot raise enough funds in the capital market to face the high liquidity shock. Instead, liquidity must be planned in advance.

Government bonds provide outside liquidity to the corporate sector, and firms use part of the proceeds of the sale of their financial claims in order to purchase government bonds in period 0. The firm can then use these bonds to absorb the liquidity shock, selling them as needed in period 1. The firm buys ℓ government bonds at date 0, and can continue at a scale i_H in the high shock state, if it satisfies the liquidity constraint

$$\left(\rho_H - \rho_0\right) i_H \leq \ell. \tag{3}$$

The corporate sector's long term investment creates enough inside liquidity (in the form of tradable rights at $t = 1$) to cope with the liquidity needs. Thus, although we have assumed that there are government bonds, our results in the idiosyncratic shocks case would hold without outside liquidity.[2] As a result of the excess supply of liquidity, the price of government bonds is driven down to 1.

3.2 Adverse selection

In this section we examine the situation in which entrepreneurs are privately informed about the quality of their projects. When outside investors cannot observe the type of the entrepreneur, then the symmetric information allocation is no longer an equilibrium outcome. A contract which implemented the symmetric information allocation for good entrepreneurs would also be selected by bad entrepreneurs and would yield negative profits for outside investors.

By Lemma 1, an equilibrium contract may be denoted just by a triplet (I, i_L, i_H); the entrepreneur receives a payment $I - A$ in date 0, receives the amount $\rho_L i_L$ or $\rho_H i_H$ in date 1, and makes the payment $\rho_0 i_L$ or $\rho_0 i_H$ on date 2. We will denote the contract offered by an entrepreneur of type T by

$$C^T \equiv \left(I^T, i_L^T, i_H^T\right)$$

where $T \in \{G, B\}$. The expected profit of each type of entrepreneur is given by

$$\pi^{id}\left(C^T; T\right) = f_T\left(\rho_1 - \rho_0\right) i_L^T + \left(1 - f_T\right)\left(\rho_1 - \rho_0\right) i_H^T - A$$

In equilibrium, the contracts being offered can influence outside investors' beliefs as to the type of entrepreneur with whom he is contracting. We use the intuitive criterion

[2] Government bonds will provide actual benefits in the aggregate shocks case discussed below.

(Cho and Kreps, 1987) as our equilibrium refinement concept. Depending on parameter values, as we will show, there can be a separating or a pooling outcome in equilibrium.

A separating contract C offered by the good entrepreneurs, satisfies the following conditions:

- It is a contract that is profitable for outside investors—that is it satisfies their participation constraint

$$f_G\left(\rho_0-\rho_L\right)i_L+\left(1-f_G\right)\left(\rho_0-\rho_H\right)i_H\geq I-A. \tag{4}$$

- Good entrepreneurs find the contract profitable

$$\pi^{id}\left(C;G\right)\geq 0. \tag{5}$$

- Bad entrepreneurs do not find it profitable

$$\pi^{id}\left(C;B\right)\leq 0. \tag{6}$$

If the good entrepreneurs offer a separating contract in equilibrium, then the bad entrepreneurs will not succeed in investing, because their project is not profitable. Thus the only candidate equilibrium separating contract will be the one which maximizes the payoff of the good entrepreneurs among all contracts satisfying conditions (4), (5) and (6).[3]

A pooling contract C offered by both good and bad entrepreneurs, satisfies the following conditions:

[3] It is also a contract which satisfies the intuitive criterion. This criterion eliminates all the separating equilibria except for the most efficient one, since dominated separating equilibria have unreasonable off-the-equilibrium path beliefs.

- It is a contract that is profitable for outside investors—that is it satisfies their participation constraint

$$\overline{f}\left(\rho_0-\rho_L\right)i_L+\left(1-\overline{f}\right)\left(\rho_0-\rho_H\right)i_H\geq I-A.$$

- Good and bad entrepreneurs find the contract profitable, that is

$$\pi^{id}\left(C;T\right)\geq 0$$

for $T=G,B$.

- Good entrepreneurs have no temptation to signal their type so as to reduce their funding costs. Good entrepreneurs can signal their type by offering a contract $\overline{C}$ different from the equilibrium pooling contract.[4] Formally, there does not exist $\overline{C}$ which is profitable if offered only to good entrepreneurs and for which

$$\pi^{id}\left(\overline{C};G\right)>\pi^{id}\left(C;G\right)\text{ and }\pi^{id}\left(\overline{C};B\right)\leq\pi^{id}\left(C;B\right).$$

Next we characterize equilibrium in the idiosyncratic shocks case for different parameter values. We find that when the equilibrium satisfies the intuitive criterion it takes one of the following three forms: (i) separating with partial insurance, (ii) separating without insurance, and (iii) pooling without insurance. In the idiosyncratic case, pooling with partial insurance or separating with full insurance is never an equilibrium. In the following subsections we establish the criteria for each of these cases, and Figure 1 shows the different equilibria of the model as a function of the probabilities of the low

[4] The gain from signaling their type depends on the off-the-equilibrium path beliefs of outside investors. The intuitive criterion specifies that outside investors do not revise their convictions and retain their prior beliefs if they observe an unexpected contract offer by an entrepreneur, unless the profit of the bad type is reduced with the new contract being offered (and in which the entrepreneur is perceived as being the good type). If the profit of the bad entrepreneurs is reduced with the unexpected contract being offered, then outside investors update their beliefs and place zero probability on the contract being offered by a bad entrepreneur.

liquidity shock by good and bad projects for particular values of the other parameters of the model.

3.2.1 Severe inefficiency of bad entrepreneurs (partial insurance regime)

Proposition 2 *(Separating equilibrium with partial insurance) The unique equilibrium is a separating equilibrium with partial insurance if and only if* $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$. *The scale of the initial investment is greater than the investment in the symmetric information allocation.*

Area A shows the set of probabilities f_B and f_G such that the conditions in Proposition 2 hold. The appendix describes the equilibrium contract in greater detail. It is important to note the scale of the initial investment in this case is greater than the investment in the symmetric information allocation and the project is then downsized in the high liquidity shock case. Good entrepreneurs partially liquidate their projects when they suffer a high liquidity shock because they obtain partial liquidity insurance, that is they set $i_H < I$.

Since bad projects are relatively inefficient, it becomes cheap for good entrepreneurs to signal their type by getting partial liquidity insurance. Liquidation when the firm suffers a high liquidity shock is more onerous for bad entrepreneurs, since this state is more likely for them. The differing cost structure allows good entrepreneurs to signal their type by downsizing their projects in the high liquidity shock state.

The condition of the proposition describes the threshold value of f_B at which bad entrepreneurs are indifferent between investing or not, when they are offered a no-insurance contract under the terms that would be offered to good entrepreneurs. If this contract is not tempting to the bad entrepreneurs, then it follows that the good entrepreneur need not set $i_H = 0$ to signal his type.

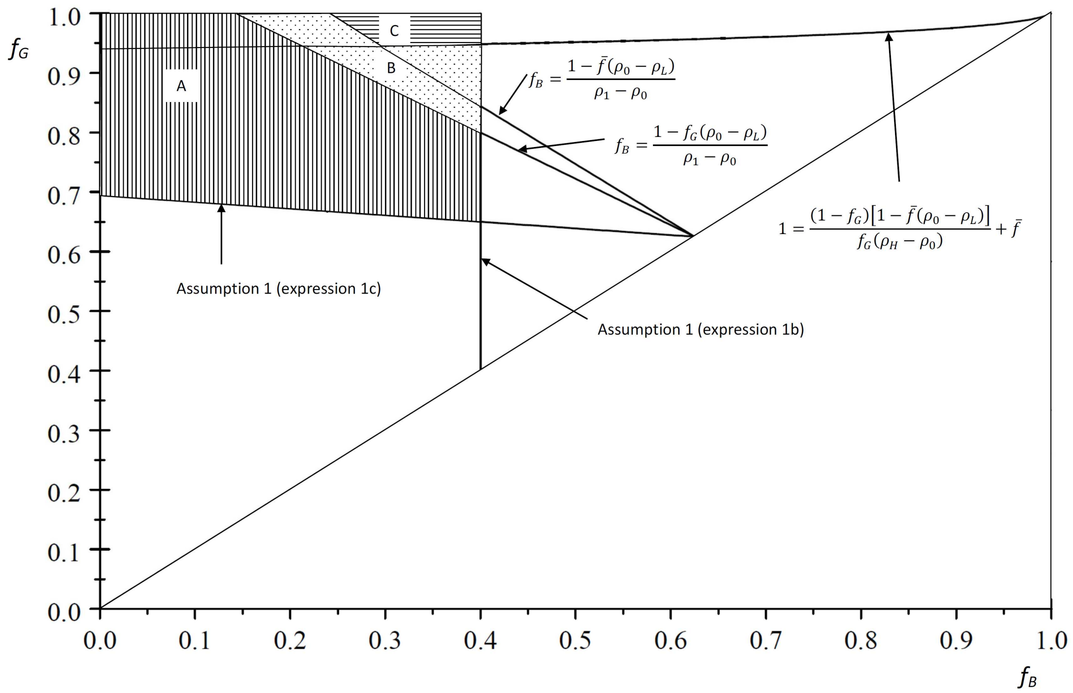

Figure 1: Equilibrium in idiosyncratic shocks case as a function of the probabilities of low liquidity shock among types of entrepreneur. Area A represents separating equilibria with partial insurance, areas B and C represent separating equilibria without insurance, and area C represents pooling equilibria without insurance. Other parameters in this example are $\rho_1 = 1.6, \rho_H = 1, \rho_0 = 0.9, \rho_L = 0$, and $\alpha = 0.9$.

Because the insurance is incomplete, the ex post shadow value of liquidity is different across firms: those with high liquidity shocks place greater value on additional liquidity ex post. However, as in the symmetric information case, there is no role for outside liquidity to alleviate this shortage. Ex post liquidity-short firms are unable to obtain additional liquidity because they have reached the limits of their pledgeable income. Ex ante, there is no demand for additional liquidity because of firms' desires to signal their types. Since the net supply of liquidity ex ante is positive, the price of government bonds q is driven down to 1.

3.2.2 Mild inefficiency of bad entrepreneurs (liquidity freeze regime)

Proposition 3 *(Separating equilibrium without liquidity insurance). There is a separating equilibrium without insurance if and only if* $f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.

Since bad firms are relatively more efficient than in the equilibrium described in Proposition 2, it becomes more difficult for good entrepreneurs to signal their type. In this case, setting $i_H = 0$ is insufficient for good entrepreneurs to separate from bad entrepreneurs. Therefore, in order to signal their type they also liquidate part of their initial investment in the low liquidity shock case ("burn money").[5]

In this equilibrium the corporate sector does not obtain any liquidity insurance since $i_H = 0$. The corporate sector uses no liquidity ex post, because firms which have high liquidity shocks close down completely. Any liquidity provided externally is absorbed by consumers.

The appendix sets out the conditions for the uniqueness of equilibrium. Area B in Figure 1 shows the set of parameter values for which there is a unique equilibrium,

[5] Firms downsize their projects in the low liquidity shock state—a state in which they have plenty of liquidity. In the aftermath of the crisis, many European banks and big US firms piled up large amounts of idle liquidity. No single factor explains companies and banks' high savings, but excess liquidity holdings do signal the strength of their balance sheets.

and area C shows the set of parameters where there is separating equilibrium without insurance but which is not the unique equilibrium. In this area, it is also possible to have a situation in which all entrepreneurs (good and bad) invest. In this case, like the previous case, there is no liquidity insurance.

Proposition 4 *(Pooling equilibrium without liquidity insurance). There is a pooling equilibrium without insurance if and only if* $f_B > \frac{1-\overline{f}(\rho_0-\rho_L)}{\rho_1-\rho_0}$ *and* $1 > \frac{(1-f_G)\left[1-\overline{f}(\rho_0-\rho_L)\right]}{f_G(\rho_H-\rho_0)} + \overline{f}$. *There is no pooling equilibrium with liquidity insurance.*

In this equilibrium, good entrepreneurs find pooling less expensive than the costs of signalling for a separating equilibrium. In the pooling equilibrium the agents continue the firm at full size in the low liquidity shock state; instead the good entrepreneurs suffer a lemons penalty from the fact that outside investors value their projects at the population average.

As the first condition indicates, this pooling outcome occurs when the bad entrepreneurs have low probabilities of suffering a high liquidity shock (that is, they are only slightly worse risks than the good entrepreneurs). The second condition requires that the price of liquidity (equal to 1) is higher than its shadow value, so that entrepreneurs prefer to liquidate in the high shock state.

A pooling equilibrium with liquidity insurance does not satisfy the intuitive criterion. When the pooling contract offers insurance, a good entrepreneur can credibly signal his type by reducing i_H a little, and obtain a gain from the reduction in the cost of funding.

When the pooling equilibrium exists, a separating equilibrium without insurance also exists. The following corollary compares their efficiency.

Corollary 1 *The pooling equilibrium without liquidity insurance Pareto dominates the* *separating equilibrium without liquidity insurance.*

This is because the signaling constraint lies "inside" the participation constraint of outside investors with pooling.

3.2.3 Conclusions

There are two regimes in the financial market with idiosyncratic shocks. In the first regime, it is cheap for good entrepreneurs to signal their type so that they get partial liquidity insurance. In the second regime, liquidity does not flow from liquidity-long to liquidity-short firms and projects with positive social value are terminated.

The intuition for the change in regimes is the following. Good entrepreneurs signal their quality by liquidation in the high liquidity shock state. When bad projects are very inefficient, signalling only requires partial liquidation.

As bad projects become less inefficient, even complete liquidation in the high shock state is insufficient to discriminate good from bad borrowers. Either good entrepreneurs must signal by liquidating their project both in the high shock state and in the low shock state, or signaling becomes so expensive that good entrepreneurs would rather pool with the bad type.

Note that a reduction in the pledgeable income ρ_0 tightens the participation constraint of outside investors, thus aggravating the adverse selection problem. The pledgeable income ρ_0 might be interpreted as the value of collateral. During the subprime and the European crises, many borrowers were unable to obtain liquidity in money markets. In some cases, these difficulties were preceded by large falls in the value of the collateral backing their loans.

The model also illustrates how adverse selection can disrupt the implementation of monetary policy. The ex post shadow value of liquidity differs across firms although the market price of liquidity is the same for all firms. Injecting additional liquidity will not solve the inefficiencies.

4 Aggregate liquidity shocks

The previous section showed the liquidity injection is not effective in improving the allocation of liquidity. It might be argued, though, that this is unsurprising since in the model without adverse selection Holmström and Tirole show that outside liquidity plays no role in the idiosyncratic shocks case. Therefore in this section we consider the aggregate liquidity shocks case, in which Holmström and Tirole show that the provision of outside liquidity becomes useful.

In the aggregate shocks case, the corporate sector is unable to generate liquidity to cope with high liquidity shocks. With perfectly correlated liquidity shocks, it is impossible to redistribute liquidity within the corporate sector ex post. Even if entrepreneurs wanted to buy insurance against high liquidity shocks, no firm would be able to offer such a contract since all firms suffer the same shock.

In order to model the problem of aggregate liquidity shocks with adverse selection, consider three states, one in which both types of firms have low liquidity shocks $\{\rho_L \rho_L\}$, one in which both types have high shocks $\{\rho_H \rho_H\}$ and, finally, one state in which good firms have low shocks and bad firms have high shocks $\{\rho_L \rho_H\}$. Thus bad entrepreneurs are identifiable when their firms suffer a large shock, and the good entrepreneurs receive a low shock—that is, in state $\{\rho_L \rho_H\}$. We assume that state $\{\rho_L \rho_L\}$ occurs with probability f_B, $\{\rho_H \rho_H\}$ with probability $1 - f_G$, and $\{\rho_L \rho_H\}$ with probability $f_G - f_B$.

As before, we assume that on average firms are socially useful but are not self-financing, but bad firms are not socially useful. These conditions are guaranteed by Assumption 1. Expression (1b) implies outside investors will not finance bad firms if they identify them. Hence, bad firms are not funded in state $\{\rho_L \rho_H\}$ because, (i) ex ante, these projects have negative net present value, and outside investors do not insure them in state $\{\rho_L \rho_H\}$ and, (ii) ex post, bad firms will not be able to obtain finance in state $\{\rho_L \rho_H\}$, as they have insufficient pledgeable income.

Assumption 1 implies

$$\frac{1+[f_B+(f_G-f_B)\,\alpha]\,\rho_L+(1-f_G)\,\rho_H}{f_B+(f_G-f_B)\,\alpha+f_G} < \frac{1+[f_B+(f_G-f_B)\,\alpha]\,\rho_L}{f_B+(f_G-f_B)\,\alpha} < \rho_1. \tag{7}$$

The two fractions in this expression represent the expected costs of one unit of the average project (adjusted by the probability of completing the project), when (i) good projects are never abandoned and (ii) all projects are abandoned in state $\{\rho_H \rho_H\}$, respectively. The term $f_G - f_B$ is multiplied by α because outside investors only finance a measure α of good firms in state $\{\rho_L \rho_H\}$. The above expression implies that the average project is socially useful (that is, α is large enough), and it is better to continue the average project in state $\{\rho_H \rho_H\}$. Recall that expression (1a) implies *a fortiori* that the average project is not self-financing.

The provision of outside liquidity Let i_L and i_H represent the continuation scales in states $\{\rho_L \rho_L\}$ and $\{\rho_H \rho_H\}$, respectively, and let i_{LH} represent the continuation scale in state $\{\rho_L \rho_H\}$.

In the absence of outside liquidity, all entrepreneurs must liquidate their projects in state $\{\rho_H \rho_H\}$. Government bonds provide outside liquidity to the corporate sector, and they are the only source of liquidity insurance in state $\{\rho_H \rho_H\}$ since consumers cannot make promises on their future income. With outside liquidity, firms (or the intermediary) can hoard liquid securities that can be resold when needed. The firm buys ℓ government bonds at date 0, and can continue at a scale i_H in state $\{\rho_H \rho_H\}$, if it satisfies the liquidity constraint $(\rho_H - \rho_0)\, i_H \leq \ell$. Without loss of generality, we assume that bad firms return their liquidity ℓ to outside investors in state $\{\rho_L \rho_H\}$.

The authorities control the price of liquidity q. Recall that the price of government bonds is denoted by q. In the idiosyncratic shocks case, the price of government bonds is always equal to 1 in equilibrium. In the aggregate shocks case, however, the price of government bonds q can be above 1. The existence of liquidity at a price $q > 1$ makes investment i_H comparatively more expensive.

We start our analysis with the symmetric information case and, later, analyze the case with adverse selection. As in the previous section, there is a regime in which firms seek partial insurance and a regime in which firms do not seek any insurance. Additionally, there is a regime in which it is possible to implement the symmetric information allocation.

4.1 Symmetric information about the type of the firm (no adverse selection)

Under symmetric information, the equilibrium allocation is the same as in the idiosyncratic shocks case when $q = 1$, but a liquidity premium may exist with aggregate shocks. When liquidity is too expensive, good entrepreneurs do not insure and end up liquidating their projects in the high liquidity shock state. The condition for continuation in the high liquidity shock state is $q < \overline{q}$, where

$$\overline{q} \equiv 1 + \frac{(1 - f_G)\left[1 - f_G\left(\rho_H - \rho_L\right)\right]}{f_G\left(\rho_H - \rho_0\right)}.$$

The parameter $\overline{q}$ is the threshold price at which good entrepreneurs are indifferent between continuing or not when the firm suffers a high liquidity shock. The value $\overline{q}$ is calculated by equating the expected profit with continuation in state $\{\rho_H \rho_H\}$ to the expected profit with termination in this state.

Proposition 5 *(Symmetric information contracts). Under symmetric information about the quality of the firms, bad entrepreneurs' projects are not funded. When $q < \overline{q}$, there is a unique equilibrium in which good entrepreneurs invest*

$$\frac{A}{1 - f_G(\rho_0 - \rho_L) - (q - f_G)(\rho_0 - \rho_H)}$$

and obtain full liquidity insurance $(i_L = i_H = I)$. When $q > \overline{q}$, there is unique equilibrium in which good entrepreneurs invest

$$\frac{A}{1 - f_G(\rho_0 - \rho_L)}$$

and obtain no liquidity insurance ($i_L = I$ and $i_H = 0$).

4.2 Adverse selection

By Lemma 1, an equilibrium contract may be denoted by a 4−tuple (I, i_L, i_{LH}, i_H); the entrepreneur receives a payment $I - A$ in date 0, receives the amount $\rho_L i_L$, $\rho_L i_{LH}$ or $\rho_H i_H$ in date 1, and makes the payment $\rho_0 i_L$, $\rho_0 i_{LH}$ or $\rho_0 i_H$ on date 2. We will denote the contract offered by an entrepreneur of type T by

$$C^T \equiv \left(I^T, i_L^T, i_{LH}^T, i_H^T\right)$$

where $T \in \{G, B\}$ and $i_{LH}^B = 0$.[6] The expected profit of each type of entrepreneur is given by

$$\pi^{ag}\left(C^T; T\right) = f_T\left(\rho_1 - \rho_0\right) i_L^T + \left(f_G - f_B\right)\left(\rho_1 - \rho_0\right) i_{LH}^T + \left(1 - f_T\right)\left(\rho_1 - \rho_0\right) i_H^T - A.$$

Given the government's choice of a price for outside liquidity, an *equilibrium* is characterized by a choice of contract for each type of entrepreneur. We apply the intuitive criterion as our equilibrium refinement concept, and there can be a separating or a pooling outcome in equilibrium.

A separating contract C proposed by the good entrepreneurs satisfies the following conditions:

- Outside investors are willing to accept the contract

$$f_B\left(\rho_0 - \rho_L\right) i_L + \left(f_G - f_B\right)\left(\rho_1 - \rho_0\right) i_{LH} + \left(1 - f_G\right)\left(\rho_0 - \rho_H\right) i_H \geq I - A + (q - 1)\,\ell.$$

- Good entrepreneurs must be willing to accept the contract

$$\pi^{ag}\left(C; G\right) \geq 0.$$

- The contract is not profitable to bad entrepreneurs

$$\pi^{ag}\left(C; B\right) \leq 0. \tag{8}$$

A pooling contract C offered by good and bad entrepreneurs satisfies the following conditions:

[6] The optimal pooling contract has the continuation scales identical for good and bad entrepreneurs in those states in which they cannot be identified. Ex post, it is possible to distinguish the type of the entrepreneur in state $\{\rho_L \rho_H\}$, so that the optimal pooling contract requires that bad entrepreneurs liquidate their project in this state whereas good entrepreneurs can set $i_{LH}^G > 0$.

- The contract satisfies the participation constraint of outside investors

$$f_B(\rho_0 - \rho_L) i_L + (f_G - f_B)(\rho_1 - \rho_0) i_{LH}\alpha + (1 - f_G)(\rho_0 - \rho_H) i_H \geq I - A + (q - 1)\ell.$$

- Good and bad entrepreneurs find the contract profitable

$$\pi^{ag}(C; T) \geq 0$$

for $T = G, B$.

- Good entrepreneurs have no temptation to signal their type so as to reduce their funding costs. Formally, there does not exist $\overline{C}$ which is profitable if offered only to good entrepreneurs and for which

$$\pi^{ag}\left(\overline{C}; G\right) > \pi^{ag}(C; G) \text{ and } \pi^{ag}\left(\overline{C}; B\right) \leq \pi^{ag}(C; B).$$

Next we characterize equilibrium in the aggregate shocks case for different parameter values. We find that when the equilibrium satisfies the intuitive criterion it takes the same three forms as in the idiosyncratic shocks case: (i) separating with partial insurance, (ii) separating without insurance, and (iii) pooling without insurance. Moreover, in the aggregate shocks case, unlike the idiosyncratic shocks case, there are parameter values under which the symmetric information allocation can be implemented even with adverse selection. Define

$$\overline{\overline{q}} = 1 - \frac{1 - f_B(\rho_1 - \rho_L) - (f_G - f_B)(\rho_0 - \rho_L) - (1 - f_G)(\rho_1 - \rho_H)}{\rho_H - \rho_0}.$$

The symmetric information contract with full insurance is not profitable to bad entrepreneurs when $q > \overline{\overline{q}}$.

In the following subsection we establish the conditions for each of these cases.

4.2.1 Types of equilibrium contracts

The next three propositions cover the major cases which can arise under adverse selection. The remaining case, which is less interesting, is relegated to the appendix. These cases depend on the size of the price of liquidity q, and on the probabilities of low liquidity shock for good and bad firms f_G and f_B.

Proposition 6 *(Separating equilibrium with partial insurance) The unique equilibrium is a separating equilibrium with partial insurance if* $q < \min\left\{\overline{q}, \overline{\overline{q}}\right\}$ *and* $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.

The first condition is equivalent to the statement that both good and bad entrepreneurs prefer full insurance. Since bad entrepreneurs are relatively inefficient, good entrepreneurs avoid paying a lemons premium by signaling their type with partial liquidation in state $\{\rho_H \rho_H\}$. The second condition is the same condition as in the idiosyncratic shocks case, and guarantees that the separating contract without insurance is not attractive to bad entrepreneurs.

Proposition 7 *(Separating equilibrium without insurance) There is a separating equilibrium without insurance if* $f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$ *and* $q \notin \left(\overline{\overline{q}}, \overline{q}\right)$.

Good firms do not obtain liquidity insurance and, ex post, liquidate their projects if they suffer a high liquidity shock. Still, the separating contract without insurance attracts bad entrepreneurs, so that signaling the good type requires setting $i_H = 0$ and downsizing investment in state $\{\rho_L \rho_L\}$—a state in which all firms have plenty of liquidity. Financial markets are not willing to provide unsubsidized insurance in state $\{\rho_H \rho_H\}$.

In addition to the separating equilibrium without insurance, it is also possible to have a pooling equilibrium without liquidity insurance. This equilibrium exists only if

$f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$, $q \notin (\overline{\overline{q}}, \overline{q})$ and $q > 1$, and the conditions for its existence are noted in the appendix.

In the aggregate shocks case, unlike the idiosyncratic shocks case, there are parameter values under which the symmetric information allocation can be implemented even with adverse selection.

Proposition 8 *(Symmetric information allocation) The unique equilibrium is the symmetric information equilibrium with insurance if* $q \in (\overline{\overline{q}}, \overline{q})$. *The unique equilibrium is the symmetric information equilibrium without insurance if* $q > \overline{q}$ *and* $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.

4.2.2 Aggregate demand for liquidity

The shape of the aggregate demand for liquidity schedule is:

- Decreasing in the full insurance case.
- Increasing in the partial insurance case. An increase in the price of liquidity tightens the participation constraint of outside investors, and forces the good entrepreneur to move along the signaling constraint (8). As a result, the demand for liquidity increases as its price increases.
- Zero in the liquidity freeze case.

The appendix presents a complete description of the aggregate demand for liquidity.

A key feature of the equilibrium (relevant for economic policy) is that reducing the price of liquidity q may reduce the demand for liquidity.

Figure 2 depicts the aggregate demand schedule for liquidity in the adverse selection case when $1 < \overline{\overline{q}} < \overline{q}$ and $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$ (there is no demand for liquidity when $f_B >$

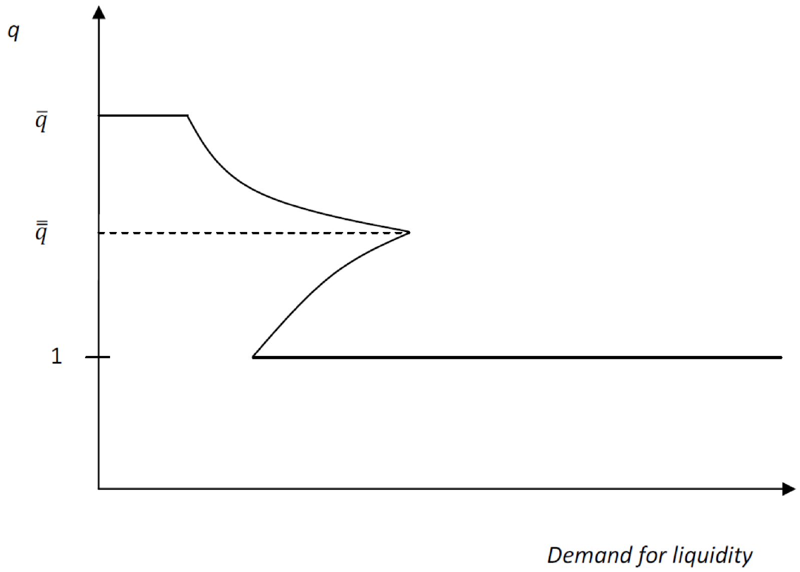

Figure 2: Liquidity demand in the aggregate shocks case, when $1 < \bar{\bar{q}} < \bar{q}$ and $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.

$\frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$). As the figure shows, reducing the price of liquidity q to the lower bound (which is equivalent to flooding the market with outside liquidity) may aggravate the adverse selection problem. When liquidity is expensive $\left(\bar{\bar{q}} < q < \bar{q}\right)$, markets achieve the symmetric information with full insurance and there is no ex post volatility in output. Reducing the price of liquidity attracts bad entrepreneurs. Good entrepreneurs distort their credit contracts so as to distinguish themselves, which entails partial insurance with ex post volatility in output.

Figure 2 illustrates a key dilemma faced by those authorities whose single policy tool is manipulating the price of liquidity q. On the one hand, fixing $q = 1$ does not solve the adverse selection problem. On the other hand, setting a positive liquidity premium leads the economy to the symmetric information equilibrium where entrepreneurs are fully insured against liquidity shocks (when $\bar{\bar{q}} < q < \bar{q}$) but also leads to an inefficient low level of initial investment.

5 Economic policies and moral hazard

In order to study the role of economic policies, we follow a mechanism design approach which relies on market mechanisms. We allow government authorities to fix taxes and subsidies which transfer resources to and from firms. The combination of taxes and subsidies can be interpreted as the reduced form version of a wider set policy tools. The optimal combination between taxes and subsidies is endogenous to the constraints of the economic environment.

We consider three instruments which affect the pledgeable income of the corporate sector: (*i*) a contingent subsidy rate s, which is transferred when firms suffer a high liquidity shock, (*ii*) a tax rate t on initial investment, and (*iii*) a subsidy rate S which raises the return on capital for entrepreneurs (but not for outside investors or consumers; Appendix A.9 presents the case in which this subsidy also affects outside investors).[7]

We do not consider taxes and subsidies on nonpledgeable income. If the public authorities were able to tax nonpledgeable income, and use these taxes to subsidize pledgeable income, then the authorities would be able to effectively overcome the fundamental constraint of our model: that not all income is pledgeable.[8]

When liquidity flows throughout the economy, the best policy is to inject liquidity such that the price of liquidity equals 1. The key policy tool to deal with adverse selection is the subsidy S, which raises the opportunity cost of investment so as to exclude bad projects from financial markets, thereby reducing the adverse selection problem.

On top of the adverse selection problem there is moral hazard. Ex post, it is always

[7] One could consider a more tailored tax strategy as, for example, a contingent tax rate which is paid when the firm suffers a low liquidity shock. This might be useful in a more complex environment, but does not provide additional advantages in our simple environment. We assume that the government is able to prevent consumers from accessing subsidies.

[8] As in Holmström and Tirole (1998, 2013), if the central planner were able to transform nonpledgeable income into pledgeable income, then the central planner would be able to implement the first-best.

optimal to rescue firms that did not get insurance and suffered high liquidity shocks. Since the initial investment is a sunk cost and $\rho_1 > \rho_H$, it is not efficient to close down these firms. One possible ex post policy would be to provide a subsidy equal to $\rho_H - \rho_0$, and let outside investors lend an amount equal to the pledgeable income ρ_0.

Such bailout policy creates moral hazard at the initial date, as entrepreneurs would anticipate ex post interventions and would not get liquidity insurance ex ante. With bailouts, the insurance market unravels at the initial date.[9]

We contrast the bailout case with the case in which public authorities have the ability to commit (at the initial date) to a policy in which they can (credibly) promise not to bail out firms.

5.1 Idiosyncratic liquidity shocks

The objective function of good entrepreneurs contains the nonpledgeable income—which taxes and subsidies cannot affect—as well as the cost of capital—which may be affected by the subsidy S. The participation constraint of outside investors consists of pledgeable income, and this income can be taxed or subsidized. Good entrepreneurs

[9] Recent research has emphasized the importance of time-consistent policies. These policies are likely to be more relevant in the aggregate shocks case. In practical terms, it is easier for public authorities to credibly commit not to rescue firms when the number of failures is small, since welfare losses are small and failed firms may be acquired by the surviving firms. Yet, the lack of commitment in the aggregate shocks case creates implicit bailout guarantees, thus inducing firms to correlate the risk inherent in their investment choices. In this case, firms have incentives to herd ex ante so as to increase the likelihood of being rescued ex post, and public authorities should use *macro*prudential policies which induce firms to differentiate their risks.

solve the following problem in a separating equilibrium.

$$\max_{\{I,i_L,i_H\}} f_G(\rho_1-\rho_0)\,i_L+(1-f_G)(\rho_1-\rho_0)\,i_H-(1+S)\,A \tag{9}$$

subject to

$$f_G(\rho_0-\rho_L)\,i_L+(1-f_G)(\rho_0-\rho_H+s)\,i_H-tI\geq I-A \tag{10}$$

$$f_B(\rho_1-\rho_0)\,i_L+(1-f_B)(\rho_1-\rho_0)\,i_H-(1+S)\,A\leq 0 \tag{11}$$

$$0\leq i_L,i_H\leq I.$$

Public authorities can manipulate the opportunity cost of the endowment A, thus shifting zero-isoprofit line for bad entrepreneurs and the cost of signalling for good entrepreneurs. The subsidy S allows the government to effectively identify and separate good from bad borrowers, without incurring the distortionary signaling costs which hurt good entrepreneurs.

The government is able to achieve the symmetric information allocation—with full liquidity insurance and investment equal to I^*. Both the participation constraint of outside investors and the signaling constraint bind at the optimal policy, which can be implemented in the following way:

- Use the signaling constraint (11) to calibrate the subsidy S, so as to achieve the optimal level of investment.
- Use the participation constraint of outside investors (10) to set the tax rate t and the contingent subsidy rate s.

Proposition 9 *(Optimal policy) The government can implement the symmetric information allocation with a subsidy* $S=\frac{\rho_1-\rho_0}{1-f_G(\rho_0-\rho_L)-(1-f_G)(\rho_0-\rho_H)}-1>0$. *The tax and the contingent subsidy must satisfy* $t=(1-f_G)\,s$.

Given the law of large numbers, the value of the tax collected is equal to the value of the contingent subsidy. The optimal policy requires a net transfer of resources equal to SA to bad entrepreneurs; good entrepreneurs do not receive this transfer, as they carry on with their projects.

Having solved the adverse selection problem, we can address the moral hazard problem. Depending on the ability of the government to commit to ex post policies, one of two natural solutions arise.

Time-consistent policies with ex post bailout An ex post bailout policy is equivalent to setting a bailout subsidy rate s equal to $\rho_H - \rho_0$. The optimal policy leads the government to set a tax rate $t = (1 - f_G)(\rho_H - \rho_0)$. Given the law of large numbers, the value of the tax is equal to the value of the bailouts. The moral hazard and the adverse selection problems are separable since the bailout policy has no implications for the subsidy S. The ex post bailout can replicate the first-best, with the following features:

- The government provides liquidity insurance.
- The bailout policy prevents financial markets from working properly.

Policies with commitment The government commits not to bail out firms and good entrepreneurs solve program (9), in which the subsidy s might be different from the bailout subsidy (per unit of investment) $\rho_H - \rho_0$. Without loss of generality, let $t = s = 0$.

Entrepreneurs need to buy liquidity insurance, and the cost of continuation in the high liquidity shock state is borne by outside investors (wherein the government bears the whole cost of continuation with bailouts).[10]

[10] Entrepreneurs do not want to "resell" their subsidies to consumers as their gain from continuation, $\rho_1 - \rho_0$, is larger than the value of the subsidy.

In this case, policies with commitment can implement the symmetric information allocation, with the following features:

- Financial markets continue to function, providing liquidity insurance.
- When compared with the bailout solution, public authorities do not need to provide taxes or contingent subsidies.

5.2 Aggregate liquidity shocks

The corporate sector in unable to insure high liquidity shocks, and the best policy is to create outside liquidity. There are two forms of outside liquidity. First, the government provides complete insurance to firms by taxing consumers. In this case, the government implements a bailout policy by transferring income from consumers to firms in the high shock state. Second, the government supplies government bonds, and firms (or the intermediary) hoard liquid securities that can be resold when needed.

The optimal provision of liquidity The government can provide liquidity by promoting the use of government bonds. The larger the supply of government bonds, the lower the price of liquidity q. The optimal provision of government bonds depends on the cost of providing this form of liquidity.

As long as there is no cost in providing outside liquidity beyond the opportunity cost of capital, then the optimal policy prescribes setting the price of liquidity equal to 1. A positive liquidity premium $q-1$ would lead to an inefficient level of investment.

Adverse selection and moral hazard Good entrepreneurs solve the following problem in a separating equilibrium.

$$\max_{\{I,i_L,i_H,i_{LH},\ell\}} f_B(\rho_1-\rho_0)\,i_L+(f_G-f_B)(\rho_1-\rho_0)\,i_{LH}$$
$$+(1-f_G)(\rho_1-\rho_0)\,i_H-(1+S)\,A$$

subject to

$$f_B(\rho_0-\rho_L)\,i_L+(f_G-f_B)(\rho_0-\rho_L)\,i_{LH}$$
$$+(1-f_G)(\rho_0-\rho_H+s)\,i_H-tI\geq I-A+(q-1)\,\ell$$
$$(\rho_H-\rho_0-s)\,i_H\leq\ell$$
$$f_B(\rho_1-\rho_0)\,i_L+(1-f_G)(\rho_1-\rho_0)\,i_H-(1+S)\,A\leq 0$$
$$0\leq i_L,i_H,i_{LH}\leq I$$

The government is able to achieve the optimal symmetric information allocation. The optimal policy can be implemented in the following way:

- The liquidity premium is zero.
- The optimal subsidy S equals $\frac{(f_B+1-f_G)(\rho_1-\rho_0)}{1-f_G(\rho_0-\rho_L)-(1-f_G)(\rho_0-\rho_H)}-1$.
- The tax and contingent subsidy rates satisfy $t=(1-f_G)\,s$.

The optimal policies with and without commitment by the government share similar features with the idiosyncratic shocks case. The optimal time-consistent policy with ex post bailout sets $s=\rho_H-\rho_0$ and $t=(1-f_G)(\rho_H-\rho_0)$.

6 Conclusion

This paper's primary aim is to consider the limits to the flow of liquidity under adverse selection. We analyze the allocation of liquidity among firms with heterogeneous liquidity shocks. The efficient allocation of liquidity requires channeling funds from liquid to illiquid firms, thus making the ex post shadow value of liquidity equal across projects.

However, the existence of a set of firms with bad projects prevents financial markets from performing the efficient allocation of liquidity. Firms will voluntarily self-ration their use of liquidity so as to signal their type, thus leading to the inability to distribute liquidity efficiently ex post.

The model shows the limits to aggregate liquidity policies in those circumstances. When firms self-ration their use of liquidity, then policies which create an excess supply of liquidity have little impact on liquidity demand. We analyze alternative policies which rebuild the liquidity channels throughout the economy, and show that the key policy tool is a subsidy which affects the opportunity cost of investment by entrepreneurs, since it impinges on the willingness of bad entrepreneurs to invest in their own projects.

The optimal time-consistent policy provides ex post liquidity insurance to institutions; since this insurance can create moral hazard problems, preemptive restrictions are also part of the policy mix. The liquidity insurance can come in the form of lower interest rates, reducing the quality of the assets accepted as collateral with low haircuts in loans or repurchase agreements, the purchase of illiquid assets at favorable terms, extending a variety of debt guarantees to financial institutions. The preemptive restrictions can come in the form of restrictions on debt, capital and liquidity.

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A Appendix A

The proof of Lemma 1 is available on request from the authors. For convenience in later calculations, we define $\Omega = 1 - \overline{f}(\rho_0 - \rho_L)$ with $0 < \Omega < 1$ by Assumption 1 (expression 2).

A.1 Proof of Proposition 2

In a separating equilibrium, good entrepreneurs solve the following problem.

$$\max_{\{I,i_L,i_H\}} f_G(\rho_1 - \rho_0) i_L + (1 - f_G)(\rho_1 - \rho_0) i_H - A \tag{12}$$

subject to

$$f_G(\rho_0 - \rho_L) i_L + (q - f_G)(\rho_0 - \rho_H) i_H \geq I - A \tag{13}$$

$$f_B(\rho_1 - \rho_0) i_L + (1 - f_B)(\rho_1 - \rho_0) i_H - A \leq 0 \tag{14}$$

$$0 \leq i_L, i_H \leq I$$

Since the signaling constraint (14) binds, one can write i_H as a function of i_L. Substitute the signaling constraint into maximization problem, and rewrite the problem as

$$\max_{\{I,i_L\}} \left(f_G - \frac{(1-f_G) f_B}{1-f_B}\right)(\rho_1 - \rho_0) i_L + \left(\frac{1-f_G}{1-f_B} - 1\right) A$$

subject to

$$\left[f_G(\rho_0 - \rho_L) - \frac{(q-f_G) f_B}{1-f_B}(\rho_0 - \rho_H)\right] i_L + \left[\frac{(q-f_G)(\rho_0 - \rho_H)}{(1-f_B)(\rho_1 - \rho_0)} + 1\right] A \geq I$$

$$0 \leq i_L, \frac{A}{(1-f_B)(\rho_1 - \rho_0)} - \frac{f_B}{1-f_B} i_L \leq I.$$

Since the objective function and the left-hand side of the participation constraint of outside investors are increasing in i_L, then i_L should be as large as possible. As a result, one of the two constraints, $i_L \leq I$ or $0 \leq \frac{A}{(1-f_B)(\rho_1-\rho_0)} - \frac{f_B}{1-f_B} i_L \Leftrightarrow i_L \leq \frac{A}{f_B(\rho_1-\rho_0)}$ must bind, and $i_L = \min\left\{\frac{A}{f_B(\rho_1-\rho_0)}, I\right\}$.

Lemma 2 *$i_L = I$ and $i_H > 0$, when $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.*

Since $i_L = I$, and the signaling constraint and the participation constraint of outside investors bind, the separating contract lies at the intersection between both constraints. As a result,

$$I = \frac{(q - f_G)(\rho_0 - \rho_H) + (1 - f_B)(\rho_1 - \rho_0)}{(1 - f_B)(\rho_1 - \rho_0)\left[1 - f_G(\rho_0 - \rho_L) + (q - f_G)(\rho_0 - \rho_H)\frac{f_B}{1-f_B}\right]} A$$

with $i_H = -\frac{f_B}{1-f_B} I + \frac{A}{(1-f_B)(\rho_1-\rho_0)}$. In this case:

1. Good entrepreneurs make positive profit. The profit of good entrepreneurs equals $\Pi_S = f_G(\rho_1 - \rho_0) i_L + (1 - f_G)(\rho_1 - \rho_0) i_H - A$, and Assumption 1 and $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$ guarantee $\Pi_S > 0$ when $q = 1$.
2. Good entrepreneurs do not want to deviate from the separating equilibrium, as they would be perceived as bad entrepreneurs and obtain zero profit.

Other separating equilibria entail good entrepreneurs playing strictly dominated strategies and, therefore, do not satisfy the intuitive criterion. In Proposition 4 we show there is no pooling equilibrium. Hence the separating equilibrium with partial insurance is unique.

The net supply of inside liquidity of the corporate sector $f_G(\rho_0 - \rho_L) I + (1 - f_G)(\rho_0 - \rho_H) i_H$ is positive since $f_G(\rho_0 - \rho_L) I + (q - f_G)(\rho_0 - \rho_H) i_H = I - A > 0$ for any $q > 1$, so that $q = 1$.■

A.2 Proof of Proposition 3

Repeating the initial steps in the proof of Proposition 2, $i_L = \min\left\{\frac{A}{f_B(\rho_1-\rho_0)}, I\right\}$.

Lemma 3 *$i_L = \frac{A}{f_B(\rho_1-\rho_0)}$ and $i_H = 0$, when $f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.*

The separating contract lies at the intersection between the signaling constraint and the $i_H = 0$ line. From the participation constraint of outside investors (13),

$$f_G(\rho_0 - \rho_L)\frac{A}{f_B(\rho_1 - \rho_0)} \geq I - A \Leftrightarrow \frac{f_G(\rho_0 - \rho_L) + f_B(\rho_1 - \rho_0)}{f_B(\rho_1 - \rho_0)}A \geq I$$

Since there is infinite supply of outside capital, the participation constraint of outside investors binds, so that the above expression holds with equality. This implies inefficient liquidation in the low shock state.

It is easy to show that:

1. Good entrepreneurs make positive profit. Profit with separation equals $\Pi_S = f_G(\rho_1 - \rho_0)i_L - A = \left(\frac{f_G}{f_B} - 1\right)A$
2. Good entrepreneurs do not want to deviate from the separating equilibrium, as they would be perceived as bad entrepreneurs and thus make zero profit.

Other separating equilibria do not satisfy the intuitive criterion. The aggregate demand for liquidity by the corporate sector is nil, since there is complete liquidation in the high shock state. All liquidity is absorbed by consumers, so that $q = 1$.■

A.3 Proof of Proposition 4

In a pooling equilibrium, good entrepreneurs solve the following problem.

$$\max_{\{I,i_L,i_H\}} f_G\left(\rho_1-\rho_0\right)i_L+\left(1-f_G\right)\left(\rho_1-\rho_0\right)i_H-A \tag{15}$$

$$subject \quad to$$

$$\overline{f}\left(\rho_0-\rho_L\right)i_L+\left(q-\overline{f}\right)\left(\rho_0-\rho_H\right)i_H\geq I-A \tag{16}$$

$$0\leq i_L,i_H\leq I.$$

Variable i_L has a positive impact on the objective function, and since high values of i_L have no impact on the participation constraint (16), then $i_L = I$. Replace i_L with I in the maximization problem, and the objective function indicates that the entrepreneur wants to set I as high as possible. Assumption 1 (expression 2) guarantees that the participation constraint of outside investors binds, and one obtains the investment function. Replacing the investment function in the objective function of good entrepreneurs, and defining $\Pi\left(x,q\right)$ as the expected profit of the entrepreneur as a function of $x=\frac{i_H}{I}$ and the market price of liquidity q, that is

$$\Pi\left(x,q\right)=\left(\frac{\left[f_G+\left(1-f_G\right)x\right]\left(\rho_1-\rho_0\right)}{1-\overline{f}\left(\rho_0-\rho_L\right)-\left(q-\overline{f}\right)\left(\rho_0-\rho_H\right)x}-1\right)A.$$

Since the problem is linear, $i_H\in\{0,I\}$ and it suffices to compare $\Pi\left(0,q\right)$ with $\Pi\left(1,q\right)$. Hence, the entrepreneur sets $i_H=I$ if and only if $\Pi\left(0,q\right)\leq\Pi\left(1,q\right)$ and we obtain the condition for continuation in the high shock state $q\leq\widehat{q}\equiv\frac{\left(1-f_G\right)\Omega}{f_G\left(\rho_H-\rho_0\right)}+\overline{f}$. Yet, there is no pooling equilibrium with $q<\widehat{q}$, since the the good entrepreneur would rather pool with insurance and the next lemma states there is no pooling equilibria with $i_H>0$.

Lemma 4 *There is no pooling equilibrium with liquidity insurance.*

Proof. A pooling equilibrium with insurance does not satisfy the intuitive criterion. When the pooling contract offers insurance, a good entrepreneur can offer a separating contract "along" the zero-isoprofit line of bad entrepreneurs $\pi^{id}(C; B) = 0 \Leftrightarrow f_B(\rho_1 - \rho_0) i_L + (1 - f_G)(\rho_1 - \rho_0) i_H - A = 0$. The good entrepreneur can signal his type by reducing i_H a little, and obtain a gain from the reduction in the cost of funding. Only pooling equilibria with $i_H = 0$ may satisfy the intuitive criterion. ■

Consider the remaining case $\widehat{q} < q = 1$, so that $i_H = 0$. For the existence of a pooling equilibrium, two conditions must be fulfilled:

1. Both types of entrepreneur want to pool, that is the participation constraints of good and bad entrepreneurs are satisfied. Since $f_B > \frac{1-\overline{f}(\rho_0-\rho_L)}{\rho_1-\rho_0}$, the participation constraint of bad entrepreneurs

$$\pi^{id}\left(\frac{A}{1-\overline{f}(\rho_0-\rho_L)}, \frac{A}{1-\overline{f}(\rho_0-\rho_L)}, 0; B\right) \geq 0$$

is satisfied. The participation constraint of good entrepreneurs is satisfied, since it is looser than the participation constraint of bad entrepreneurs.

2. Good entrepreneurs do not want to deviate from the pooling equilibrium. They cannot make a profit with a contract that bad entrepreneurs reject. The pooling contract features $i_H = 0$; since the isoprofit lines of good entrepreneurs are steeper than those of bad entrepreneurs, then any deviating contract along the isoprofit line of the bad entrepreneurs will reduce the profit of the good entrepreneur. Hence, there is no profitable deviation for the good entrepreneurs, and we have a pooling equilibrium.

The aggregate demand for liquidity by the corporate sector is nil, since there is complete liquidation in the high shock state. All liquidity is absorbed by consumers, so that $q = 1$.■

A.4 Proof of Proposition 6

In a separating equilibrium, good entrepreneurs solve the following problem.

$$\max_{\{I, i_L, i_{LH}. i_H\}} f_B\left(\rho_1-\rho_0\right) i_L+\left(f_G-f_B\right)\left(\rho_1-\rho_0\right) i_{LH}+\left(1-f_G\right)\left(\rho_1-\rho_0\right) i_H-A$$

$$\text{subject to} \tag{17}$$

$$f_B\left(\rho_0-\rho_L\right) i_L+\left(f_G-f_B\right)\left(\rho_0-\rho_L\right) i_{LH}+$$
$$\left(q-f_G\right)\left(\rho_0-\rho_H\right) i_H \geq I-A \tag{18}$$

$$f_B\left(\rho_1-\rho_0\right) i_L+\left(1-f_G\right)\left(\rho_1-\rho_0\right) i_H-A \leq 0 \tag{19}$$

$$0 \leq i_L, i_{LH}, i_H \leq I.$$

The value of i_{LH} should be as high as possible, so that $i_{LH}=I$ and the participation constraint of outside investors binds. The signaling constraint also binds. If it did not, then the good entrepreneurs would choose the full insurance perfect information allocation since $q<\overline{q}$, which attracts bad entrepreneurs when $q<\overline{\overline{q}}$.

Lemma 5 *$i_L=I$ and $i_H>0$, when $f_B<\frac{1-f_G\left(\rho_0-\rho_L\right)}{\rho_1-\rho_0}$.*

Solve problem (17) by solving the participation and the signaling constraints (18) and (19), which yields

$$\begin{aligned}
i_H &= \frac{1-f_G\left(\rho_0-\rho_L\right)-f_B\left(\rho_1-\rho_0\right)}{\left(\rho_1-\rho_0\right)\left\{\left(1-f_G\right)\left[1-f_G\left(\rho_0-\rho_L\right)\right]+f_B\left(q-f_G\right)\left(\rho_0-\rho_H\right)\right\}} A \\
I &= \frac{\left(q-f_G\right)\left(\rho_0-\rho_H\right)+\left(1-f_G\right)\left(\rho_1-\rho_0\right)}{\left(\rho_1-\rho_0\right)\left\{\left(1-f_G\right)\left[1-f_G\left(\rho_0-\rho_L\right)\right]+f_B\left(q-f_G\right)\left(\rho_0-\rho_H\right)\right\}} A .
\end{aligned}$$

The good entrepreneur sets i_L and i_{LH} as large as possible, so that $i_L=i_{LH}=I$. The good entrepreneur distorts the symmetric information contract such that $i_H<I$. Profit equals $\Pi_S=\left(f_G-f_B\right)\left(\rho_1-\rho_0\right) I$.

1. The contract yields non-negative profits to good entrepreneurs, as long as $I \geq 0$.

The denominator in the expression for I is positive since $f_G + \frac{[1-f_G(\rho_0-\rho_L)](1-f_G)}{f_B(\rho_H-\rho_0)} > \overline{q} > q$. The numerator is positive as long as $q < 1 + \frac{(\rho_1-\rho_H)(1-f_G)}{\rho_H-\rho_0} = \widetilde{\widetilde{q}}$; it is easy to show that $\overline{q} < \widetilde{\widetilde{q}}$.

2. Good entrepreneurs do not want to deviate from the separating equilibrium, as they would be perceived as bad entrepreneurs and obtain zero profit.■

A.5 Proof of Proposition 7

Good entrepreneurs solve problem (17), the signaling and the budget constraints bind, and $i_{LH} = I$.

Lemma 6 *$i_L = \frac{A}{f_B(\rho_1-\rho_0)}$ and $i_H = 0$, when $f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.*

Hence, $i_L = \frac{A}{f_B(\rho_1-\rho_0)} < I = \frac{\rho_1-\rho_L}{(\rho_1-\rho_0)[1-(f_G-f_B)(\rho_0-\rho_L)]}A$ and profit equals $\Pi_S = \frac{(f_G-f_B)(\rho_1-\rho_L)}{1-(f_G-f_B)(\rho_0-\rho_L)}A$. The rest of the proof follows the same steps as the proof of Proposition 6. The profit with separation equals $\Pi_S = \frac{(f_G-f_B)(\rho_1-\rho_L)}{1-(f_G-f_B)(\rho_0-\rho_L)}A$.

1. The contract yields non-negative profits to good entrepreneurs, since $\Pi_S > 0$.

2. Good entrepreneurs do not want to deviate from the separating equilibrium, as they would be perceived as bad entrepreneurs and obtain zero profit.■

A.6 Pooling equilibrium in aggregate shocks case

We present the proposition showing the existence of the pooling equilibrium in the aggregate shocks case (not shown in the main text). Define $\widehat{\widehat{q}} \equiv \frac{(1-f_G)\Omega}{f_G(\rho_H-\rho_0)} + f_G$, with $\widehat{\widehat{q}} > 1$ and $\widehat{\widehat{q}} > \overline{q}$.

Proposition 10 *(Pooling equilibrium without insurance) There is a pooling equilibrium without insurance if and only if*

$$\begin{aligned} f_B &> \frac{1 - f_G(\rho_0 - \rho_L)}{\rho_1 - \rho_0} \\ q &\notin \left(\overline{\overline{q}}, \overline{q}\right) \\ q &> \widehat{\widehat{q}} \\ \frac{(f_G - f_B)(\rho_1 - \rho_L)}{1 - (f_G - f_B)(\rho_0 - \rho_L)} &< \frac{f_G(\rho_1 - \rho_0)}{\Omega} - 1 \\ \frac{1-\alpha}{\alpha} &< \frac{f_G(\rho_1 - \rho_0)}{\Omega} - 1. \end{aligned}$$

There is no pooling equilibrium with liquidity insurance.

The proof of the proposition is as follows. As in the idiosyncratic shocks case, no pooling equilibrium with $i_H > 0$ exists.

Lemma 7 *There is no pooling equilibrium with liquidity insurance.*

Proof. The proof is analogous to the proof of Lemma 4. ■

Since there is no pooling equilibrium with insurance, we consider a pooling equilibrium with $i_H = 0$ (which is equivalent to $q > \widehat{\widehat{q}}$). The shadow value of liquidity in a pooling equilibrium equals $\widehat{\widehat{q}}$, with $\widehat{\widehat{q}} > 1$ since $\frac{\Omega}{f_G(\rho_H - \rho_0)} > 1 \Leftrightarrow 1 - \overline{f}(\rho_0 - \rho_L) - f_G(\rho_H - \rho_0) > 1 - f_G(\rho_H - \rho_L) > 0$. We restrict the analysis to the relevant cases in which a pooling equilibrium is possible.

Lemma 8 *There is no pooling equilibrium without liquidity insurance when $f_B < \frac{1 - f_G(\rho_0 - \rho_L)}{\rho_1 - \rho_0}$.*

The symmetric information contract without insurance is not attractive to bad entrepreneurs. Hence, good entrepreneurs do not need to pool. We thus consider exclusively the cases when $f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$ and $q \notin \left(\overline{\overline{q}}, \overline{q}\right)$.

Profit in a pooling contract with $i_H = 0$ equals $\Pi_P = \left(\frac{f_G(\rho_1-\rho_0)}{1-f_B(\rho_0-\rho_L)-(f_G-f_B)(\rho_0-\rho_L)\alpha} - 1\right) A$. Good entrepreneurs compare the profit Π_P with the profit from a deviation to a contract in which their type is revealed. Hence, the pooling contract competes with a deviating contract.

Lemma 9 *The competing deviating contract has $i_H = 0$.*

Proof. The intuitive criterion implies that the good entrepreneur does not profit from moving to a separating contract along the isoprofit line of the bad entrepreneur. Profit in a deviating contract along the isoprofit of the bad entrepreneur is the result of the following problem:

$$
\begin{aligned}
&\max_{\{I, i_L, i_{LH}.i_H\}} f_B\left(\rho_1-\rho_0\right) i_L + \left(f_G - f_B\right)\left(\rho_1-\rho_0\right) i_{LH} + \left(1-f_G\right)\left(\rho_1-\rho_0\right) i_H - A \\
&\textit{subject to} \\
&f_B\left(\rho_0-\rho_L\right) i_L + \left(f_G - f_B\right)\left(\rho_0-\rho_L\right) i_{LH} + \left(q - f_G\right)\left(\rho_0-\rho_H\right) i_H \geq I - A \\
&f_B\left(\rho_1-\rho_0\right) i_L + \left(1-f_G\right)\left(\rho_1-\rho_0\right) i_H - A = \\
&\qquad\qquad f_B\left(\rho_1-\rho_0\right) \frac{A}{1-f_B\left(\rho_0-\rho_L\right)-\left(f_G-f_B\right)\left(\rho_0-\rho_L\right)\alpha} - A \\
&0 \leq i_L, i_{LH}, i_H \leq I.
\end{aligned}
$$

Replacing the "isoprofit" constraint in the objective function and it is obvious that the good entrepreneur wishes to set $i_{LH} = I$, and investment I as large as possible. Looking into the participation constraint of outside investors, it is also obvious that the best way to increase investment is by setting i_L as large as possible and $i_H = 0$. ■

We have shown that maximum profit with a deviating contract happens for $i_H = 0$. It is thus sufficient to compare pooling and deviating contracts with $i_H = 0$. Profit with pooling without insurance equals $\left(\frac{f_G(\rho_1-\rho_0)}{1-f_B(\rho_0-\rho_L)-(f_G-f_B)(\rho_0-\rho_L)\alpha} - 1\right) A$, and profit with separation without insurance equals $\frac{(f_G - f_B)(\rho_1 - \rho_L)}{1-(f_G - f_B)(\rho_0 - \rho_L)} A$. The condition $\frac{(f_G - f_B)(\rho_1 - \rho_L)}{1-(f_G - f_B)(\rho_0 - \rho_L)} < \frac{f_G(\rho_1 - \rho_0)}{\Omega} - 1$ in Proposition 10 guarantees that there is no profitable deviation from the pooling contract.

It remains to show that it is not possible to have a pooling equilibrium when $q \in \left(\overline{\overline{q}}, \overline{q}\right)$. If $q < \overline{q}$, then there would be a profitable deviation with $i_L > 0$ for the good entrepreneur. If $q > \overline{\overline{q}}$ then bad entrepreneurs do not sign the full information contract.

Finally, there is the constraint $\frac{1-\alpha}{\alpha} < \frac{f_G(\rho_1 - \rho_0)}{\Omega} - 1$ since the bad entrepreneur makes negative profit otherwise.■

A.7 Proof of Proposition 8

First, consider the symmetric information allocation with perfect insurance.

Proposition 11 *(Symmetric information allocation with perfect insurance) The unique equilibrium is the symmetric information equilibrium with insurance iff* $\overline{\overline{q}} < q < \overline{q}$.

We start with the following lemma.

Lemma 10 *Bad entrepreneurs do not sign the symmetric information contract with perfect insurance when* $q > \overline{\overline{q}}$.

The ex ante shadow value of an initial unit of liquidity (in the symmetric information equilibrium) can be calculated to be $\overline{q} = 1 + \frac{(1-f_G)[1-f_G(\rho_H - \rho_L)]}{f_G(\rho_H - \rho_0)} = f_G + \frac{(1-f_G)[1-f_G(\rho_0 - \rho_L)]}{f_G(\rho_H - \rho_0)}$. When $\overline{q} > q$, good entrepreneurs prefer the perfect insurance to the no insurance contract. Hence, the market equilibrium implements the symmetric information contract with perfect insurance when $\overline{q} > q > \overline{\overline{q}}$. For $q \notin \left(\overline{\overline{q}}, \overline{q}\right)$ it is not possible to have the full information perfect insurance contract.

Second, consider the symmetric information allocation without insurance.

Proposition 12 *(Symmetric information allocation without insurance) The unique equilibrium is the symmetric information equilibrium without insurance iff* $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$ *and* $q > \overline{q}$.

We start with a lemma.

Lemma 11 *Bad entrepreneurs do not sign the symmetric information contract without insurance when* $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.

When $q > \overline{q}$, good entrepreneurs want to set $i_H = 0$. Hence, the market equilibrium implements the symmetric information contract without insurance when $q > \overline{q}$ and $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$. It is not possible to implement the symmetric information contract without insurance if $q < \overline{q}$ and $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$ as good entrepreneurs want to set $i_H > 0$.

Lemma 12 *The symmetric information equilibrium without insurance is not an equilibrium when* $f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$.

When $f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$, bad entrepreneurs want to sign the symmetric information equilibrium without insurance.

Together, Propositions 11 and 12 prove Proposition 8.■

A.8 Aggregate demand of liquidity

Other shapes for the aggregate demand schedule are possible. When $f_B < \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$ and:

- $\overline{q} < 1$ the aggregate demand for liquidity by the corporate sector is nil.

- $\overline{\overline{q}} < 1 < \overline{q}$ the aggregate demand schedule is downward sloping for $q > 1$, and is infinitely elastic for $q = 1$.

- $1 < \overline{q} < \overline{\overline{q}}$ the aggregate demand schedule is downward sloping for $1 < q < \overline{q}$, nil for $q > \overline{q}$, and infinitely elastic for $q = 1$.

When $f_B > \frac{1-f_G(\rho_0-\rho_L)}{\rho_1-\rho_0}$ and:

- $1 < \overline{q} < \overline{\overline{q}}$ the demand for liquidity is nil for $q > 1$, and is infinitely elastic for $q = 1$.

- $1 < \overline{\overline{q}} < \overline{q}$ and the conditions specified in Propositions 7, 10 and 12, the demand for liquidity is nil for $1 < q < \overline{\overline{q}}$ or $\overline{q} < q$, is decreasing when $\overline{\overline{q}} < q < \overline{q}$ (equals the demand under symmetric information with perfect insurance), and is infinitely elastic for $q = 1$.

A.9 Non-targeted tool S: results when the opportunity cost of capital S affects outside investors

The adverse selection and the moral hazard problems are not separable anymore. For simplicity, we consider only the separating equilibrium.

A.9.1 Idiosyncratic shocks case

The problem of good entrepreneurs in a separating equilibrium can be rewritten as

$$\begin{aligned}
&\max_{\{I,i_L,i_H\}} f_G\left(\rho_1-\rho_0\right)i_L+\left(1-f_G\right)\left(\rho_1-\rho_0\right)i_H-\left(1+S\right)A \\
&subject \quad to \qquad\qquad (20)\\
&f_G\left(\rho_0-\rho_L\right)i_L+\left[\left(1+S\right)q-f_G\right]\left(\rho_0-\rho_H+s\right)i_H\geq\left(1+S\right)\left[\left(1+t\right)I-A\right] \\
&f_B\left(\rho_1-\rho_0\right)i_L+\left(1-f_B\right)\left(\rho_1-\rho_0\right)i_H-\left(1+S\right)A\leq 0 \\
&0\leq i_L,i_H\leq I.
\end{aligned}$$

Regarding the optimal policy, this specification for the impact of the opportunity cost does not affect the optimal value of S since this specification does not affect the signaling constraint. Yet, t and s must be adjusted since the value of S affects the budget constraint. Given the efficient level of investment and the efficient value of S

$$\begin{aligned}
I=I^* &= \frac{A}{1-f_G\left(\rho_0-\rho_L\right)-\left(1-f_G\right)\left(\rho_0-\rho_H\right)} \\
S &= \frac{\rho_1-\rho_0}{1-f_G\left(\rho_0-\rho_L\right)-\left(1-f_G\right)\left(\rho_0-\rho_H\right)}-1
\end{aligned}$$

the values of t and s must satisfy the budget constraint

$$f_G\left(\rho_0-\rho_L\right)I+\left[\left(1+S\right)q-f_G\right]\left(\rho_0-\rho_H+s\right)I=\left(1+S\right)\left[\left(1+t\right)I-A\right].$$

We consider time-consistent policies, so that $s=\rho_H-\rho_0$ and the budget constraint becomes

$$f_G\left(\rho_0-\rho_L\right)I=\left(1+S\right)\left[\left(1+t\right)I-A\right].$$

Substituting the efficient values of investment and S, yields

$$t=\left[1-f_G\left(\rho_0-\rho_L\right)-\left(1-f_G\right)\left(\rho_0-\rho_H\right)\right]\left(1+\frac{f_G\left(\rho_0-\rho_L\right)}{\rho_1-\rho_0}\right)-1$$

and the tax rate t could be negative (in which case we would have a subsidy).

A.9.2 Aggregate shocks case

The problem of good entrepreneurs in the aggregate shocks case is the following.

$$
\begin{aligned}
&\max_{\{I, i_L, i_H, i_{LH}, \ell\}} f_B(\rho_1 - \rho_0)\, i_L + (f_G - f_B)(\rho_1 - \rho_0)\, i_{LH} \\
&\qquad + (1 - f_G)(\rho_1 - \rho_0)\, i_H - (1 + S) A \\
&\textit{subject to} \\
&\qquad f_B(\rho_0 - \rho_L)\, i_L + (f_G - f_B)(\rho_0 - \rho_L)\, i_{LH} \\
&\qquad + (1 - f_G)(\rho_0 - \rho_H + s)\, i_H \geq (1 + S)\left[(1 + t) I - A + (q - 1)\ell\right] \\
&(\rho_H - \rho_0 - s)\, i_H \leq \ell \\
&f_B(\rho_1 - \rho_0)\, i_L + (1 - f_G)(\rho_1 - \rho_0)\, i_H - (1 + S) A \leq 0 \\
&0 \leq i_L, i_H, i_{LH} \leq I
\end{aligned}
$$

The only difference to the targeted subsidy is in the budget constraint. Considering time-consistent policies and rearranging the budget constraint,

$$f_B(\rho_0 - \rho_L)\, i_L + (f_G - f_B)(\rho_0 - \rho_L)\, i_{LH} = (1 + S)\left[(1 + t) I - A\right].$$

Since the efficient level of investment equals $\frac{A}{1 - f_G(\rho_0 - \rho_L) - (q - f_G)(\rho_0 - \rho_H)}$, then $S = \frac{(1 + f_B - f_G)(\rho_1 - \rho_0)}{1 - f_G(\rho_0 - \rho_L) - (1 - f_G)(\rho_0 - \rho_H)} - 1$ for $q = 1$, and

$$t = \left[1 - f_G(\rho_0 - \rho_L) - (1 - f_G)(\rho_0 - \rho_H)\right]\left[1 + \frac{f_G(\rho_0 - \rho_L)}{(1 + f_B - f_G)(\rho_1 - \rho_0)}\right] - 1.$$

Again, the value of the tax rate t may be negative.